Performance comparison of sequences designed from the Hall Difference Set and Orthogonal Gold Sequences of Length 32

Jennifer Seberry
University of Wollongong, jennie@uow.edu.au

Beata J. Wysocki
University of Wollongong, bjw@uow.edu.au

Tadeusz A. Wysocki
University of Nebraska-Lincoln, wysocki@uow.edu.au

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Abstract

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Performance Comparison of Sequences designed from the Hall Difference Set and Orthogonal Gold Sequences of Length 32

Jennifer Seberry, Beata J Wysocki, Tadeusz A Wysocki
Faculty of Informatics, University of Wollongong
NSW2522, Australia
Wysocki@uow.edu.au

Abstract: In the paper we perform a performance comparison between two sets of orthogonal spreading sequences i.e. sequences based on the Hadamard matrix of order 32 constructed using the Hall difference set and orthogonal Gold sequences of length 32. Both considered sets of sequences are characterised by low peaks in the aperiodic cross-correlation functions and have also good aperiodic auto-correlation properties.

1. Introduction

It is well known, e.g. [1], that if the sequences have good aperiodic cross-correlation properties, the transmission performance can be improved for those CDMA systems where different propagation delays exist. In [2], Wysocki and Wysocki have shown that spreading sequences derived from different H-equivalent matrices [3] of Sylvester's construction have different aperiodic correlation properties, and that by choosing the appropriate H-equivalent matrix, one can significantly reduce the peaks in aperiodic cross-correlation functions for the whole set of sequences.

The lowest value of peaks in the aperiodic cross-correlation functions for the sequences derived from a Hadamard matrix H-equivalent to the Sylvester-Hadamard matrix of order $N = 32$ published in [2] is 0.4063. This result is much lower than 0.9688 for sequences derived from the Sylvester-Hadamard matrix of order $N = 32$ in its well-known canonical form. On the other hand, the value of 0.4063 is still much greater than the Levenshtein bound [4] of 0.1410 for the set of 32 sequences of order 32. Of course, the bound is derived for sets of bipolar sequences without imposed condition of orthogonality for their perfect alignment.

In [5], we proposed a set of orthogonal spreading sequences of order 32 derived from the Hall difference set (31,15,7) [6], usually referred to as $\mathbf{H}_{32\text{-}03}$ [7]. We have found through computer search that the lowest value of peaks in the aperiodic cross-correlation functions for the sequences derived from a Hadamard matrix $\mathbf{W}_{32\text{-}03}$ H-equivalent to the matrix $\mathbf{H}_{32\text{-}03}$ is 0.3750. It is still significantly higher than the Levenshtein bound but is a significant improvement compared to the best result obtained from H-equivalent Sylvester-Hadamard matrices. The value of 0.3750 can be also achieved for some sets of orthogonal Gold sequences [8] or their H-equivalents.

In the paper, we present a performance comparison in terms of BER for direct sequences CDMA (DS CDMA) systems utilizing sequences derived from the matrix $\mathbf{W}_{32\text{-}03}$ and the best of the orthogonal Gold sequences sets of length 32.

The paper is organised as follows. In section 2, we introduce the Hall difference set construction and apply it to produce a $\mathbf{H}_{32\text{-}03}$ Hadamard matrix in its canonical form and show the method to search for the matrix $\mathbf{W}_{32\text{-}03}$. In section 3, we describe the method used to generate orthogonal Gold sequences. Section 4 presents simulation results for the DS CDMA systems utilizing both sets of sequences, and the paper is concluded in section 5.

2. Sequences derived from Hadamard matrix constructed using the Hall difference set

Let α be a primitive root of 31 ($\alpha = 2$ or 3 or 5) [9]. To construct the matrix $\mathbf{H}_{32\text{-}03}$ we first create a set:

$$A = \left\{\alpha^{6j}, \alpha^{6j+3}, \alpha^{6j+5}\right\} \quad j = 0,1,2,3,4 \tag{1}$$

which in the considered case is a set of 15 integers:

$$A = \{a_1, \ldots, a_{15}\} \tag{2}$$

when taken modulo 31.
Then we create a circulant matrix $\mathbf{B}$ of order 31, with first row elements $b_{1,k}$ defined as:

$$b_{1,k} = \begin{cases} 1, & \text{if} \quad k \in A \\ -1, & \text{otherwise} \end{cases} \tag{3}$$

The matrix $\mathbf{H}_{32\text{-}03}$ is created from the matrix $\mathbf{B}$ by adding the first row and the first column containing all '1s', i.e.:

$$
\mathbf{H}_{32-03} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & b_{1,1} & b_{1,2} & \cdots & b_{1,31} \\ 1 & b_{1,31} & b_{1,1} & \cdots & b_{1,30} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & b_{1,2} & b_{1,3} & \cdots & b_{1,1} \end{bmatrix} \tag{4}
$$

The matrix $\mathbf{H}_{32\text{-}03}$ is therefore equal to [7]:

$$
\mathbf{H}_{32-03} = \left[\begin{array}{l}
\texttt{++++++++++++++++++++++++++++++++} \\
\texttt{+-++++-+-+---+--+++-----++--+-++} \\
\texttt{++-++++-+-+---+--+++-----++--+-+} \\
\texttt{+++-++++-+-+---+--+++-----++--+-} \\
\texttt{+-++-++++-+-+---+--+++-----++--+} \\
\texttt{++-++-++++-+-+---+--+++-----++--} \\
\texttt{+-+-++-++++-+-+---+--+++-----++-} \\
\texttt{+--+-++-++++-+-+---+--+++-----++} \\
\texttt{++--+-++-++++-+-+---+--+++-----+} \\
\texttt{+++--+-++-++++-+-+---+--+++-----} \\
\texttt{+-++--+-++-++++-+-+---+--+++----} \\
\texttt{+--++--+-++-++++-+-+---+--+++---} \\
\texttt{+---++--+-++-++++-+-+---+--+++--} \\
\texttt{+----++--+-++-++++-+-+---+--+++-} \\
\texttt{+-----++--+-++-++++-+-+---+--+++} \\
\texttt{++-----++--+-++-++++-+-+---+--++} \\
\texttt{+++-----++--+-++-++++-+-+---+--+} \\
\texttt{++++-----++--+-++-++++-+-+---+--} \\
\texttt{+-+++-----++--+-++-++++-+-+---+-} \\
\texttt{+--+++-----++--+-++-++++-+-+---+} \\
\texttt{++--+++-----++--+-++-++++-+-+---} \\
\texttt{+-+--+++-----++--+-++-++++-+-+--} \\
\texttt{+--+--+++-----++--+-++-++++-+-+-} \\
\texttt{+---+--+++-----++--+-++-++++-+-+} \\
\texttt{++---+--+++-----++--+-++-++++-+-} \\
\texttt{+-+---+--+++-----++--+-++-++++-+} \\
\texttt{++-+---+--+++-----++--+-++-++++-} \\
\texttt{+-+-+---+--+++-----++--+-++-++++} \\
\texttt{++-+-+---+--+++-----++--+-++-+++} \\
\texttt{+++-+-+---+--+++-----++--+-++-++} \\
\texttt{++++-+-+---+--+++-----++--+-++-+} \\
\texttt{+++++-+-+---+--+++-----++--+-++-}
\end{array}\right]
$$

The modification is achieved by taking another orthogonal $N \times N$ matrix $\mathbf{D}_N$, and the new set of sequences is based on a matrix $\mathbf{W}_N$, given by:

$$
\mathbf{W}_N = \mathbf{H}_N \mathbf{D}_N \tag{5}
$$

Of course, the matrix $\mathbf{W}_N$ is also orthogonal [2].

In [2], it has been shown that the correlation properties of the sequences defined by $\mathbf{W}_N$ can be significantly different to those of the original sequences.

A simple class of orthogonal matrices of any order are diagonal matrices with their elements $d_{i,j}$ fulfilling the condition:

$$
|d_{l,m}| = \begin{cases} 0 & \text{for} \quad l \neq m \\ k & \text{for} \quad l = m \end{cases} ; \qquad l, m = 1, \ldots, N \tag{6}
$$

with k being any non-zero real constant. That way, the matrix $\mathbf{D}_N$ satisfies the condition:

$$
\mathbf{D}_N \mathbf{D}_N = \mathbf{D}_N \mathbf{D}_N^T = k^2 \mathbf{I}_N \tag{7}
$$

where $\mathbf{I}_N$ is the $N \times N$ identity matrix.

To preserve the normalization of the sequences, the elements of $\mathbf{D}_N$, being in general complex numbers, must be of the form:

$$
d_{l,m} = \begin{cases} 0 & \text{for} \quad l \neq m \\ \exp(j\phi_l) & \text{for} \quad l = m \end{cases} ; \qquad l, m = 1, \ldots, N \tag{8}
$$

The parameters ϕ_l, $l = 1, \ldots, N$, are phase coefficients, which are real numbers with values from $[0, 2\pi)$. From the implementation point of view, the best class of sequences is the one of binary sequences, i.e. when $\phi_l = 0$ or π.

To find the best possible modifying diagonal matrix $\mathbf{D}_N$ we can do an exhaustive search of all possible bipolar sequences of length N, and choose the one, which leads to the best performance of the modified set of sequences. However, this approach is very computationally intensive, and even for a modest values of N, e.g. $N = 20$, it is rather impractical. Hence, other search methods, like a random search, must be considered, e.g. Monte Carlo algorithm.

In the considered case, we have performed 10,000 random drawings of binary sequences of length 32, and used them as diagonals of the modifying matrix $\mathbf{D}_{32}$. The matrix $\mathbf{W}_{32\text{-}03}$, H-equivalent to $\mathbf{H}_{32\text{-}03}$, giving the lowest maximum peaks in any pair of aperiodic cross-correlation functions is obtained when the diagonal of the matrix $\mathbf{D}_{32}$ is

$$
[\texttt{-+-+-++++---+--+++-+++++++++++++}]
$$

The matrix $\mathbf{W}_{32\text{-}03}$ is then:

$$
\mathbf{W}_{32-03} = \left[\begin{array}{l}
\texttt{-+-+-++++---+--+++-+++++++++++++} \\
\texttt{---+-+-+--++--+-++------++--+-++} \\
\texttt{-+++-++-++-+-+---+-+-----++--+-+} \\
\texttt{-+---+++--+--+++---++-----++--+-} \\
\texttt{---+++++++-++++-+-++++-----++--+} \\
\texttt{-+++--+++-+---+--++-+++-----++--} \\
\texttt{-----+-++--+++-------+++-----++-} \\
\texttt{--+++++-+-----++--++--+++-----++} \\
\texttt{-++---++----++--+-+-+--+++-----+} \\
\texttt{-+--++-+++--+-++-++--+--+++-----} \\
\texttt{---++-+-+-+-+---+-----+--+++----} \\
\texttt{--++---+---++--+-+++---+--+++---} \\
\texttt{--+--+--++----+++---+---+--+++--} \\
\texttt{--+-+++---+-++-+++++-+---+--+++-} \\
\texttt{--+-+-++-+-++-++++--+-+---+--+++} \\
\texttt{-++-+--++++-----++-+-+-+---+--++} \\
\texttt{-+--+---+-++++-+-+-++-+-+---+--+} \\
\texttt{-+-++------+--+++--+++-+-+---+--} \\
\texttt{---+-----+---+--+++++++-+-+---+-} \\
\texttt{--++-+---++-++++-+--++++-+-+---+} \\
\texttt{-++--++--++++-+-+--+-++++-+-+---} \\
\texttt{----++++-+++-----++++-++++-+-+--} \\
\texttt{--+++-++++++-+++----++-++++-+-+-} \\
\texttt{--+----++-++-++++-++-++-++++-+-+} \\
\texttt{-++-++--+--+-++-++-++-++-++++-+-} \\
\texttt{----+-+------++--+---+-++-++++-+} \\
\texttt{-++++--+-+--+++----+--+-++-++++-} \\
\texttt{--------+++-+-+---+++--+-++-++++} \\
\texttt{-+++++----+++-----+-++--+-++-+++} \\
\texttt{-+----+--+-+---+--+--++--+-++-++} \\
\texttt{-+-+++-+-++--+-++-+---++--+-++-+} \\
\texttt{-+-+--+-+++++++++++----++--+-++-}
\end{array}\right]
$$

The plot of the peaks in the aperiodic cross-correlation functions between any two pairs of the sequences derived from the matrix $\mathbf{W}_{32\text{-}03}$ is given in Figure 1 – the solid line. The set of sequences derived from the matrix $\mathbf{W}_{32\text{-}03}$ is characterized by the following aperiodic correlation characteristics:

$$C_{max} = 0.3750$$
$$R_{CC} = 0.9682$$
$$R_{AC} = 0.9844$$

where C_{max} denotes the maximum peak value in the magnitude of aperiodic cross-correlation functions between any pair of the sequences in the set, R_{CC} is the mean square aperiodic cross-correlation for the set of sequences [1], and R_{AC} is the mean square aperiodic auto-correlation for the set of sequences [1].

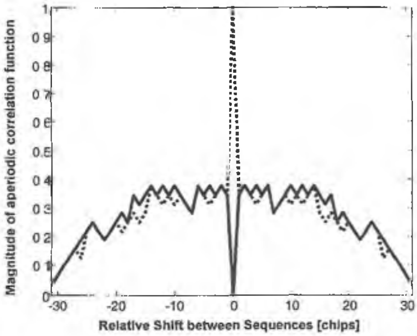

Figure 1: *Peaks in the magnitude of: aperiodic cross-correlation functions between any two pairs of the sequences derived from matrix $W_{32\text{-}03}$ – solid line, aperiodic auto-correlation functions – dotted line*

Synchronisation amenability of the derived sequences can be assessed by examining the maximum off-peak values in the magnitudes of aperiodic autocorrelation functions for the whole sequence set. From Figure 1 – the dotted line, it is visible that the sequences derived from the matrix $\mathbf{W}_{32\text{-}03}$ have a very distinctive peak for the perfect alignment and that there are no other significant peaks for any non-zero shift.

3. Orthogonal Gold sequences

The Gold sequences or Gold codes can be constructed from a preferred pair of *m*-sequences [9]. For a pair of preferred sequences $a = \{a_n\}$ and $b = \{b_n\}$ of period $N = 2^5 - 1 = 31$ the set

$$G(a,b) = \{a, b, a \oplus b, a \oplus Tb, \ldots, a \oplus T^{30} b\} \quad (9)$$

is a set of Gold sequences of order 31, where $T^k b$; $k = 1, \ldots, 30$, denotes a cyclical shift of a sequence b by k positions to the right.

The orthogonal Gold sequences $GO(a,b)$ are constructed from $G(a,b)$ by adding a zero on the first position of all elements of $G(a,b)$, and disregarding the second element, i.e. the sequence $(0,b)$ [8]. Hance, we have

$$GO_{32}(a,b) = \{(0,a), (0, a \oplus b), \ldots (0, a \oplus T^{30} b)\} (10)$$

The bipolar spreading sequences are obtained from the sequences $GO_{32}(a,b)$ through a mapping in which '0' corresponds to '1' and '1' corresponds to '-1'.
According to [9], there are 6 cyclically different *m*-sequences of period 31. These are:

$$(a_n) = (0000100101100111110001101110101)$$
$$(a_{3n}) = (0001010110100001100100111110111)$$
$$(a_{5n}) = (0011011111010001001010110000111)$$
$$(a_{7n}) = (0111110010011000010110101000111)$$
$$(a_{11n}) = (0010111110110011100001101010010)$$
$$(a_{15n}) = (0111011000111110011010010000101)$$

By choosing these sequences, (a_{3n}) and (a_{11n}) as the seed sequences in construction of orthogonal Gold sequences we obtain a set of orthogonal sequences of order 32 characterised by the following correlation parameters

$$C_{max} = 0.3750$$
$$R_{CC} = 0.9675$$
$$R_{AC} = 1.0078$$

which are almost the same as for the sequences derived from the matrix $\mathbf{W}_{32\text{-}03}$.

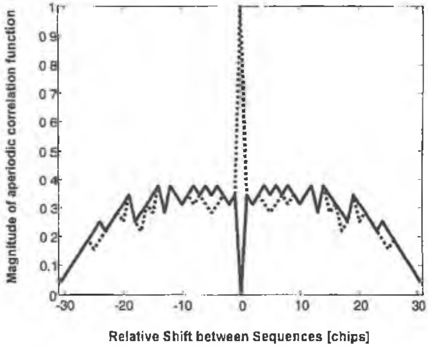

Figure 2: *Peaks in the magnitude of: aperiodic cross-correlation functions between any two pairs of the orthogonal Gold sequences derived from m-sequences, (a_{3n}) and (a_{11n})– solid line, aperiodic auto-correlation functions– dotted line.*

It should be noted here, that not all the preferred pairs of *m*-sequences lead to the same values of those correlation parameters. In fact, the value $C_{max} = 0.3750$ is the lowest value one can achieve for the orthogonal Gold codes. For the comparison with sequences derived from the $\mathbf{W}_{32\text{-}03}$ matrix, in Fig.2, we present the plots of maximum magnitudes of autocorrelation cross-correlation functions and auto-correlation functions for the constructed set of orthogonal Gold sequences.

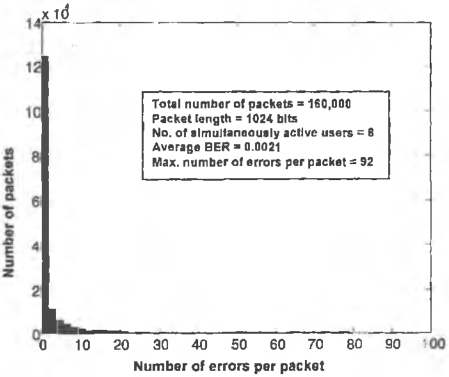

Figure 4: *Histogram of the number of errors in received packets for a system utilizing DS CDMA with BPSK and orthogonal Gold codes derived from matrix $W_{32\text{-}03}$.*

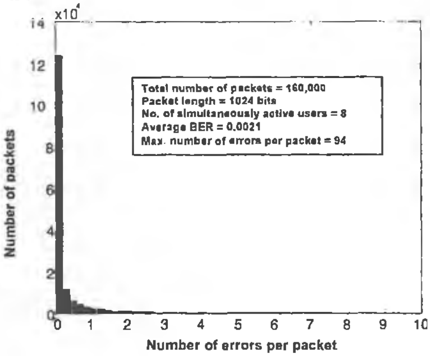

Figure 4: *Histogram of the number of errors in received packets for a system utilizing DS CDMA with BPSK and orthogonal Gold codes derived from m-sequences (a_{3n}) and (a_{11n}).*

4. Simulations

In Fig. 3 and Fig. 4, we present the simulation results for the 32 channel asynchronous DS CDMA BPSK system utilizing sequences derived from the $\mathbf{W}_{32\text{-}03}$ matrix and the orthogonal Gold sequences derived from *m*-sequences (a_{3n}) and (a_{11n}), respectively. In both cases, we had simulated the same number of 8 randomly chosen simultaneous active users, and transmitted the same number of 1024-bit frames in each of the 32 possible channels. The results have been then averaged across the 32 channels. The transmission channel was assumed to be an AWGN channel with an $E_b/N_0 = 20$ dB.

5. Conclusions

In the paper we presented the performance comparison of the orthogonal spreading sequences derived from the Hadamard matrix of order 32 ($\mathbf{W}_{32\text{-}03}$) constructed from the Hall difference set and orthogonal Gold sequences derived from *m*-sequences (a_{3n}) and (a_{11n}) [9]. It has been found that both set of sequences are characterised by almost identical aperiodic correlation parameters. In fact, the values of peaks in the aperiodic cross-correlation functions and the maximum value of the off-peak autocorrelation functions are the same for both sets of sequences. Additionally, the error performance of the DS CDMA BPSK systems utilizing sequences from both sets is also almost identical with the average BER for 8 active users being the same, i.e. 0.0021, and the maximum number of erroneous bits per 1024 packets being equal to .... And 94 for orthogonal Gold sequences.

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